

A Naturally Logical Representation for DNA Primary Sequences

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Abstract: In this paper, we (1) introduce a logical representation (LR) for DNA primary sequences; (2) show relations between LR and some other representations including the characteristic sequences of a DNA sequence, Randic's 2-D, 4-D representations, and Z-curve (a 3-D graphical representation); and (3) outline the constructions of the S/S matrix specific for a logical sequence and its 2*2 condensed matrix.

Keywords: DNA sequence, logical representation, condensed matrix, invariant.

1. INTRODUCTION

With the completion of the genome projects for human, mouse and rat one after another, and the development of some similar projects for other species, the number of known DNA sequences is increasing at an exponential rate. A DNA sequence is usually expressed in terms of a series of four letters A, G, C, and T, which denote the four nucleic acid bases: adenine, guanine, cytosine, and thymine, respectively. This expression is called a letter sequence representation or a DNA primary sequence. DNA primary sequences vary enormously in their length. They might consist of fewer than a hundred bases but also can extend to over a hundred thousand bases. Even when the long sequences are broken down into segments corresponding to exons or introns, the segments corresponding to the same position within a gene and belonging to different species may have different lengths. Clearly, this means that a base-by-base comparison of genes cannot be used. Therefore, the alignment of sequences has been considered in the literature. The standard procedures of alignment consider differences between strings due to deletion-insertion, compression-expansion, and substitution of the string elements. Finding the smallest number of changes that are necessary to match labels in two sequences is far from trivial.

In order to give a visual and hence qualitative characterization of DNA primary sequences, graphical representations were suggested [1-13], which mainly regard a DNA primary sequence as a curve embedded in 2-D plane or 3-D space. Besides these, Randic and Balaban [14] proposed a 4-D representation of DNA primary sequences by assigning to each of the four bases A, T, G, C directions along the four orthogonal coordinate axes. Although it seemed as if the four bases can be assigned in $4!=24$ ways, it is not difficult to find that there is only one essential pattern of the 4-D representation corresponding to the same DNA sequence, because the four directions of 4-D space are fully equivalent. Of course, as pointed out by the authors, the important advantage of graphical visualization of DNA sequences has been lost.

In 2002, He and Wang [15] introduced another representation for DNA sequences, which is based on the idea of the coarse-grained description of the DNA primary sequence. As we know, the four nucleic acid bases A, G, C and T can be divided into two classes according to their chemical structures, *i.e.* purine $R = \{A, G\}$ and pyrimidine $Y = \{C, T\}$. The bases can be also divided into another two classes, amino group $M = \{A, C\}$ and keto group $K = \{G, T\}$. In addition, the division can be made according to the strength of the hydrogen bond, *i.e.* weak H-bonds $W = \{A, T\}$ and strong H-bonds $S = \{G, C\}$ [11, 15-17]. By labeling the elements of R , M and W by 1, and that of Y , K and S by 0, respectively, He and Wang transform a DNA primary sequence into three (0,1)-sequences, which are named the (R, Y)-, (M, K)-, and (W, S)-characteristic sequences of the DNA sequence, respectively.

In this paper, we offer a naturally logical representation for DNA primary sequences, which is different from the existing representations of DNA sequences while it is closely related to some of them.

2. THE LOGICAL REPRESENTATION FOR DNA PRIMARY SEQUENCES

It is easy to see that the four bases A, G, C and T can be classified into two classes based on the knowledge of logic: A and not-A (G or C or T). Denoting not-A by \bar{A} , we reduce a DNA primary sequence into a binary sequence: by 1 (0) we denote A (\bar{A}). The obtained (0,1)-sequence is called the A-sequence of the DNA sequence. In this representation, some information of the DNA sequence structure may be lost; however, this will give prominence to local information from adenine.

Besides this we can similarly define other three (0,1)-sequences: G-sequence, C-sequence and T-sequence, respectively. We state the above process in mathematical terms as follows.

Let $X = x_1x_2\cdots$ be a DNA primary sequence. We define four homomorphism maps $\phi_i (i = 1, 2, 3, 4)$ by $\phi_i(X) = \phi_i(x_1)\phi_i(x_2)\cdots$, where

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gene, we list its **LR** in the order of A-T-G-C in Table 2. Suppressing 'zeros' and linking only 'ones' one after another, we immediately obtain the same 2-D graphical representation of the DNA sequence.

Table 2. The LR of the Sequence ATGGTGCACCTGACT CCTGA

LR ^A (S)	1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1
LR ^T (S)	0 1 0 0 1 0 0 0 0 0 0 1 0 0 0 1 0 0 1 0 0
LR ^G (S)	0 0 1 1 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0
LR ^C (S)	0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 1 0 1 1 0 0 0

3.3. The LR and Randic's 4-D Representation

In [14], a 4-D representation of DNA primary sequences was considered by assigning to each of the four bases A, T, G, C directions along the four orthogonal coordinate axes:

A (1, 0, 0, 0)

T (0, 1, 0, 0)

G (0, 0, 1, 0)

C (0, 0, 0, 1)

Taking the first 15 bases of the exon-1 of human *beta*-globin gene as an example, its 4-D coordinates are listed in Table 3.

Table 3. 4-D Coordinates for the First 15 Bases of the First Exon of Human *Beta*-Globin Gene^a

Human <i>Beta</i> -Globin															
No.	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15														
Base	A T G G T G C A C C T G A C T														
<i>e_A</i>	1	1	1	1	1	1	1	2	2	2	2	3	3	3	
<i>e_T</i>	0	1	1	1	2	2	2	2	2	3	3	3	3	4	
<i>e_G</i>	0	0	1	2	2	3	3	3	3	3	4	4	4	4	
<i>e_C</i>	0	0	0	0	0	1	1	2	3	3	3	3	4	4	

^aTaken from Table 2 [14].

For our purpose, by $LR_i^Y(S)$ we denote the *i*-th 'base' in the logical sequence $LR^Y(S)$, (*Y*=A, T, G, C). Define

$$\begin{cases} x_{1i} = \sum_{k=1}^i LR_k^A(S) \\ x_{2i} = \sum_{k=1}^i LR_k^T(S) \\ x_{3i} = \sum_{k=1}^i LR_k^G(S) \\ x_{4i} = \sum_{k=1}^i LR_k^C(S) \end{cases}$$

then, we immediately obtain the same 4-D coordinates (cf. Table 2).

3.4. The LR and the Z-Curve

Consider a DNA sequence read from the 5'to the 3'-end with *N* bases. Inspect it by stepping one base at a time. In the step *n* (*n* = 1, 2, ..., *N*), count the *cumulative* occurrence numbers of A, C, G and T, respectively, in the subsequence from the 1st base to the *n*-th base in the sequence inspected, and denote them by *A_n*, *C_n*, *G_n* and *T_n*, respectively. Then Zhang and Zhang [11] proposed a 3-D curve which consists of a series of nodes *P_n*, whose coordinates *x_n*, *y_n* and *z_n* are constructed as follows:

$$\begin{cases} x_n = 2(A_n + G_n) - n, \\ y_n = 2(A_n + C_n) - n, \\ z_n = 2(A_n + T_n) - n, \end{cases} \quad x_n, y_n, z_n \in [-N, N], \quad n = 0, 1, 2, \dots, N,$$

where *A*₀ = *C*₀ = *G*₀ = *T*₀ = 0 and hence *P*₀=0. The Z curve is defined as the connection of the nodes *P*₀, *P*₁, *P*₂, ..., *P_N* one by one sequentially with straight lines.

From the definition of **LR** and Table 1, it is easy to see

that $Y_n = \sum_{k=1}^n LR_k^Y(S)$, (*Y*=A, T, G, C). So, we can obtain the

above nodes *P_n*'s from the logical sequences directly, and then the Z curve.

4. THE S/S MATRIX REPRESENTATION FOR LOGICAL SEQUENCES

For a DNA primary sequence, Randic [18] introduced a *S/S* matrix by regarding the sequence itself as an input. Similarly, we consider the *S/S* matrix for the logical sequences.

Suppose that *b*₁*b*₂*b*₃...*b_n* is a (0,1)-sequence, we construct its *S/S*=(*s_{ij}*) matrix as follows:

$$s_{ji} = s_{ij} = \frac{n_{ij}}{j-i}, \quad (j>i); \quad s_{ii} = 0,$$

where *n_{ij}* is the number of the 'logical bases' which are identical to the one corresponding to *b_j* in the sub-string *b_{i+1}b_{i+2}*...*b_j*.

Note that the **LR** of a DNA sequence includes four logical sequences. Therefore, from a given DNA sequence, one can obtain four *S/S* matrices following the method above. Consider the first 20 bases of the exon-1 of human *beta*-globin gene:

A₁ T₂ G₃ G₄ T₅ G₆ C₇ A₈ C₉ C₁₀ T₁₁
G₁₂ A₁₃ C₁₄ T₁₅ C₁₆ C₁₇ T₁₈ G₁₉ A₂₀

In Table 4, we show the *S/S* matrix corresponding to its G-sequence (see LR^G(S) of Table 2).

Observing Table 4, we see that the entries next to the main diagonal in the *S/S* matrix are always equal to 1. In fact, by the definition of *S/S*, the matrix elements for all adjacent pairs of bases must equal 1 and for nonadjacent ones, the matrix elements are less than or equal to 1. Such a matrix has an interesting advantage, that is, from which one can construct a convergent sequence of matrices ^{*k*}*S/S*, (*k*=1, 2, 3, ...), whose (*i, j*)-element is *s_{ij}^k*. Clearly, as *k* trends to in-

Table 4. The Upper Triangles of the Matrix S/S Corresponding to LR^G(S)

S/S	0 ₁	0 ₂	1 ₃	1 ₄	0 ₅	1 ₆	0 ₇	0 ₈	0 ₉	0 ₁₀	0 ₁₁	1 ₁₂	0 ₁₃	0 ₁₄	0 ₁₅	0 ₁₆	0 ₁₇	0 ₁₈	1 ₁₉	0 ₂₀
0 ₁	0	1/1	1/2	2/3	2/4	3/5	3/6	4/7	5/8	6/9	7/10	4/11	8/12	9/13	10/14	11/15	12/16	13/17	5/18	14/19
0 ₂		0	1/1	2/2	1/3	3/4	2/5	3/6	4/7	5/8	6/9	4/10	7/11	8/12	9/13	10/14	11/15	12/16	5/17	13/18
1 ₃			0	1/1	1/2	2/3	2/4	3/5	4/6	5/7	6/8	3/9	7/10	8/11	9/12	10/13	11/14	12/15	4/16	13/17
1 ₄				0	1/1	1/2	2/3	3/4	4/5	5/6	6/7	2/8	7/9	8/10	9/11	10/12	11/13	12/14	3/15	13/16
0 ₅					0	1/1	1/2	2/3	3/4	4/5	5/6	2/7	6/8	7/9	8/10	9/11	10/12	11/13	3/14	12/15
1 ₆						0	1/1	2/2	3/3	4/4	5/5	1/6	6/7	7/8	8/9	9/10	10/11	11/12	2/13	12/14
0 ₇							0	1/1	2/2	3/3	4/4	1/5	5/6	6/7	7/8	8/9	9/10	10/11	2/12	11/13
0 ₈								0	1/1	2/2	3/3	1/4	4/5	5/6	6/7	7/8	8/9	9/10	2/11	10/12
0 ₉									0	1/1	2/2	1/3	3/4	4/5	5/6	6/7	7/8	8/9	2/10	9/11
0 ₁₀										0	1/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	2/9	8/10
0 ₁₁											0	1/1	1/2	2/3	3/4	4/5	5/6	6/7	2/8	7/9
1 ₁₂												0	1/1	2/2	3/3	4/4	5/5	6/6	1/7	7/8
0 ₁₃													0	1/1	2/2	3/3	4/4	5/5	1/6	6/7
0 ₁₄														0	1/1	2/2	3/3	4/4	1/5	5/6
0 ₁₅															0	1/1	2/2	3/3	1/4	4/5
0 ₁₆																0	1/1	2/2	1/3	3/4
0 ₁₇																	0	1/1	1/2	2/3
0 ₁₈																		0	1/1	1/2
1 ₁₉																			0	1/1
0 ₂₀																				0

finitly, the limit of the matrices sequence { ^kS/^kS } turns into a (0,1)-matrix, which is denoted by ^bS/^bS.

However, as we have seen, the orders of S/S and its ‘higher order’ matrices are always equal to the lengths of the sequences considered. Therefore, lengthy sequences will produce such very ‘large’ matrices that they will have to be processed somehow if useful information is to be gained. To deal with this problem, one can consider the following two aspects.

4.1. The Invariant

Once a real symmetric matrix representation of a sequence is given, some of invariants extracted from the matrix, such as the average matrix element, the average row sum, the leading eigenvalue, and the ‘ALE-index’, can be used as descriptors of the sequence [8, 9, 12-15,19-22]. For instance, the ‘ALE-index’ corresponding to the S/S matrix above is easily calculated as 14.4126, and its normalized value is 0.72063.

4.2. The Condensed Matrix

Similar to the 4×4 condensed matrix specific for the DNA primary sequence [18], we propose a 2×2 condensed matrix the elements of which are all extracted from the matrix S/S associated with a logical sequence. We present briefly the method for constructing a condensed matrix as follows.

Suppose that the logical sequence considered consists of m 1’s and n 0’s. Thus, by gathering all elements of S/S corresponding to bases 1’s, one can obtain a partitioned matrix:

	0....0	1...1
0	00 _{n×n}	01 _{n×m}
...		
1	10 _{m×n}	11 _{m×m}
...		

replacing each partition by a suitable number, a very condensed 2×2 matrix with pertinent information is formed. Various sub-matrices invariants can be selected as the ‘representative’. Of course, generally speaking, m ≠ n, therefore the (leading) eigenvalue is usually avoided.

As an example, in Table 5 we show the partitioned matrix corresponding to the S/S matrix above.

If we use the average matrix element of each sub-matrix as its ‘representative’, then a 2×2 condensed matrix can be obtained:

$$\begin{pmatrix} 0.75469 & 0.69154 \\ 0.69154 & 0.29307 \end{pmatrix}_{2 \times 2}$$

5. CONCLUSION

According to a classification of four nucleic acid bases, we introduce a naturally logical representation (LR) for DNA primary sequences. Then we compare it with some other representations. They are the characteristic sequences proposed by He and Wang [15], the 2-D ‘four horizontal lines’ graph and the 4-D representation of DNA sequences proposed by Randic *et al.* [8, 9, 14], and the Z-curve proposed by Zhang and Zhang [11]. The results show that the

Table 5. The Partitioned Matrix

0 ₁	0 ₂	0 ₅	0 ₇	0 ₈	0 ₉	0 ₁₀	0 ₁₁	0 ₁₃	0 ₁₄	0 ₁₅	0 ₁₆	0 ₁₇	0 ₁₈	0 ₂₀	l ₃	l ₄	l ₆	l ₁₂	l ₁₉
0	1/1	2/4	3/6	4/7	5/8	6/9	7/10	8/12	9/13	10/14	11/15	12/16	13/17	14/19	1/2	2/3	3/5	4/11	5/18
1/1	0	1/3	2/5	3/6	4/7	5/8	6/9	7/11	8/12	9/13	10/14	11/15	12/16	13/18	1/1	2/2	3/4	4/10	5/17
2/4	1/3	0	1/2	2/3	3/4	4/5	5/6	6/8	7/9	8/10	9/11	10/12	11/13	12/15	1/2	1/1	1/1	2/7	3/14
3/6	2/5	1/2	0	1/1	2/2	3/3	4/4	5/6	6/7	7/8	8/9	9/10	10/11	11/13	2/4	2/3	1/1	1/5	2/12
4/7	3/6	2/3	1/1	0	1/1	2/2	3/3	4/5	5/6	6/7	7/8	8/9	9/10	10/12	3/5	3/4	2/2	1/4	2/11
5/8	4/7	3/4	2/2	1/1	0	1/1	2/2	3/4	4/5	5/6	6/7	7/8	8/9	9/11	4/6	4/5	3/3	1/3	2/10
6/9	5/8	4/5	3/3	2/2	1/1	0	1/1	2/3	3/4	4/5	5/6	6/7	7/8	8/10	5/7	5/6	4/4	1/2	2/9
7/10	6/9	5/6	4/4	3/3	2/2	1/1	0	1/2	2/3	3/4	4/5	5/6	6/7	7/9	6/8	6/7	5/5	1/1	2/8
8/12	7/11	6/8	5/6	4/5	3/4	2/3	1/2	0	1/1	2/2	3/3	4/4	5/5	6/7	7/10	7/9	6/7	1/1	1/6
9/13	8/12	7/9	6/7	5/6	4/5	3/4	2/3	1/1	0	1/1	2/2	3/3	4/4	5/6	8/11	8/10	7/8	2/2	1/5
10/14	9/13	8/10	7/8	6/7	5/6	4/5	3/4	2/2	1/1	0	1/1	2/2	3/3	4/5	9/12	9/11	8/9	3/3	1/4
11/15	10/14	9/11	8/9	7/8	6/7	5/6	4/5	3/3	2/2	1/1	0	1/1	2/2	3/4	10/13	10/12	9/10	4/4	1/3
12/16	11/15	10/12	9/10	8/9	7/8	6/7	5/6	4/4	3/3	2/2	1/1	0	1/1	2/3	11/14	11/13	10/11	5/5	1/2
13/17	12/16	11/13	10/11	9/10	8/9	7/8	6/7	5/5	4/4	3/3	2/2	1/1	0	1/2	12/15	12/14	11/12	6/6	1/1
14/19	13/18	12/15	11/13	10/12	9/11	8/10	7/9	6/7	5/6	4/5	3/4	2/3	1/2	0	13/17	13/16	12/14	7/8	1/1
1/2	1/1	1/2	2/4	3/5	4/6	5/7	6/8	7/10	8/11	9/12	10/13	11/14	12/15	13/17	0	1/1	2/3	3/9	4/16
2/3	2/2	1/1	2/3	3/4	4/5	5/6	6/7	7/9	8/10	9/11	10/12	11/13	12/14	13/16	1/1	0	1/2	2/8	3/15
3/5	3/4	1/1	1/1	2/2	3/3	4/4	5/5	6/7	7/8	8/9	9/10	10/11	11/12	12/14	2/3	1/2	0	1/6	2/13
4/11	4/10	2/7	1/5	1/4	1/3	1/2	1/1	1/1	2/2	3/3	4/4	5/5	6/6	7/8	3/9	2/8	1/6	0	1/7
5/18	5/17	3/14	2/12	2/11	2/10	2/9	2/8	1/6	1/5	1/4	1/3	1/2	1/1	1/1	4/16	3/15	2/13	1/7	0

logical representation may be a useful tool for characterizing and comparing DNA sequences. To give a quantitative analysis of data on DNA sequences, we construct a S/S matrix specific for a logical sequence and its family of associated 'higher order' matrices kS^kS ($k=2,3,\dots$). From these matrices, one can derive some structurally related descriptors and the corresponding 2×2 condensed matrices.

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